Worksheet 3: Derivatives - Answers

Find y' for each of the following functions:

1. $f(x) = x^3 - 4x + 6$
   $y' = 3x^2 - 4$

2. $f(t) = \frac{1}{2}t^6 - 3t^4 + t$
   $y' = 3t^5 - 12t^3 + 1$

3. $f(u) = 6u^{-9}$
   $y' = -54u^{-10}$

4. $g(x) = 5x^{-3/5}$
   $y' = -3x^{-8/5}$

5. $y = (2x^3 + 3)(x^4 - 2x)$

$$\begin{aligned} y' &= (2x^3 + 3)\frac{d}{dx}(x^4 - 2x) + (x^4 - 2x)\frac{d}{dx}(2x^3 + 3) \\ &= (2x^3 + 3)(4x^3 - 2) + (x^4 - 2x)(6x^2) = \end{aligned}$$

6. $y = A + \frac{t}{(t-1)^2} = A + \frac{t}{t^2 - 2t + 1}$

$$\begin{aligned} y' &= \frac{dA}{dt} + \frac{(t^2 - 2t + 1)\frac{d}{dt}t - (t)\frac{d}{dt}(t^2 - 2t + 1)}{(t-1)^4} \\ &= 0 + \frac{(t^2 - 2t + 1)(1) - (t)(2t - 2)}{(t-1)^4} \end{aligned}$$

7. $y = \frac{\sin x}{x^2}$

$$y' = \frac{(x^2)\frac{d}{dx}\sin x - (\sin x)\frac{d}{dx}(x^2)}{x^4} = \frac{x^2 \cos x - 2x \sin x}{x^4}$$

8. $y = \sin x \tan x$

$$y' = \sin x \frac{d}{dx}\tan x + \tan x \frac{d}{dx}\sin x = \sin x \sec^2 x + \tan x \cos x = \sin x(\sec^2 x + 1)$$

9. $y = \frac{1 + \sin x}{x + \cos x}$

$$\begin{aligned} y' &= \frac{(x + \cos x)\frac{d}{dx}(1 + \sin x) - (1 + \sin x)\frac{d}{dx}(x + \cos x)}{(x + \cos x)^2} \\ &= \frac{(x + \cos x)(\cos x) - (1 + \sin x)(1 - \sin x)}{(x + \cos x)^2} \\ &= \frac{x \cos x + \cos^2 x - (1 - \sin^2 x)}{(x + \cos x)^2} = \frac{x \cos x}{(x + \cos x)^2} \end{aligned}$$

10. Find an equation of the tangent line to the curve $y = (2x)/(x+4)$ that is parallel to the line $2x - y = 3$.

Solving $2x - y = 3$ for y gives $y = 2x - 3$ so the slope is 2.

We need to find x such that $y' = 2$.

$$2 = y' = \frac{d}{dx}\frac{2x}{x+4} = \frac{(x+4)\frac{d}{dx}(2x) - (2x)\frac{d}{dx}(x+4)}{(x+4)^2} = \frac{(x+4)(2) - (2x)(1)}{(x+4)^2} = \frac{8}{(x+4)^2}$$
$$\implies 2(x+4)^2 = 8 \implies (x+4)^2 = 4 \implies x+4 = \pm\sqrt{4} = \pm 2 \implies x = -4 \pm 2 = -6, -2$$

At $x = -6$, $y = \frac{2(-6)}{-6+4} = \frac{-12}{-2} = 6$. Using $y = 2x + b$ gives

$$6 = 2(-6) + b = -12 + b \implies b = 18 \implies y = 2x + 18$$

At $x = -2$, $y = \frac{2(-2)}{-2+4} = \frac{-4}{2} = -2$. Using $y = 2x + b$,

$$-2 = 2(-2) + b = -4 + b \implies b = 2 \implies y = 2x + 2$$

11. Find an equation of the tangent line to the curve $y = 1/(1+x^2)$ at the point $(-1, 1/2)$

$$y' = \frac{(1+x^2)\frac{d}{dx}(1) - (1)\frac{d}{dx}(1+x^2)}{(1+x^2)^2} = \frac{(1+x^2)(0) - (1)(0+2x)}{(1+x^2)^2} = \frac{-2x}{(1+x^2)^2}$$

Hence the slope at $x = -1$ is

$$m = y'(-1) = \frac{-2(-1)}{(1+(-1)^2)^2} = \frac{2}{4} = \frac{1}{2}$$

Using $y - y_1 = m(x - x_1)$ gives

$$y - \frac{1}{2} = \frac{1}{2}(x+1) = \frac{x}{2} + \frac{1}{2} \implies y = \frac{1}{2}x + 1$$