

Warm-up: Price Demand equation

Several companies make a 37 inch, Plasma HDTV. Right now, if we charge \$1440 we can expect to sell 1000 TVs. But if we charge \$2440 we can expect to sell 500 TVs. That is, as the price goes up, the consumer demand will decrease. Assume that consumer demand depends on the price linearly. Write a price demand equation: $x=ap+b$. Solve for price.

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§11.4 Elasticity

The student will learn about:

- Relative Rate of change.**
- Elasticity of demand.**

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Motivation

A broker is trying to sell you two stocks, Biotech and Comstat. The broker estimates that Biotech's earnings will increase \$2 per year over the next year, while ComStat's earnings will increase only \$1 per year. Express these as mathematical formulas:

Is this sufficient information for you to choose between these two stocks? What other information might you request from the broker to help you decide?

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Relative and Percentage Rates of Change

FACT: $\frac{u'(x)}{u(x)}$ is called the relative rate of change of $u(x)$.

What does it measure?

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Relative and Percentage Rates of Change

The relative rate of change of $u(x)$ is $\frac{u'(x)}{u(x)}$

A model for the GDP is $f(t)=300t+6000$ where t is in years since 1990. Find the graph of the relative rate of change of $f(t)$ for $5 < t < 12$. What does it tell you?

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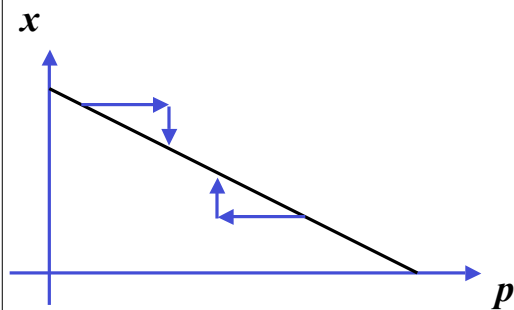
Price sensitivity of demand: Elasticity

So far we have almost always expressed price as a function of demand:

E.g. $p=p(x)=40-(1/1000)x$.

Often we can find an inverse to this function and thus express demand as a function of price:

E.g.



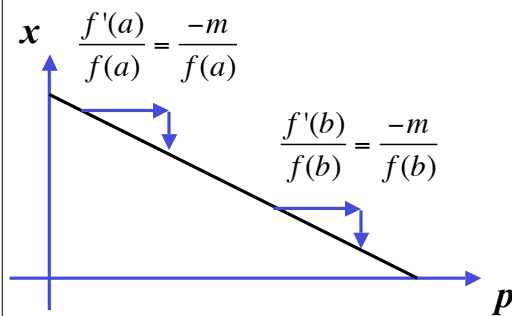
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Price sensitivity of demand: Elasticity

Economists use elasticity of demand to study the relationship between changes in price and changes in demand on a relative scale.

Elasticity of demand is the negative ratio of the relative rate of change of demand to the relative rate of change of price.

If $x=f(p)=B-mp$ then $E(p)=$



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Price sensitivity of demand: Elasticity

The Elasticity of demand if $x = f(p)$ is

$$E(p) = -\frac{pf'(p)}{f(p)} = -\frac{\text{relative rate of change of demand}}{\text{relative rate of change of price}}$$

relative rate of change of demand = $-E(p)$ (relative rate of change of price)

percent change of demand = $-E(p)$ (percent change of price)

Find $E(p)$ when $x=f(p) = 40,000-1000p$

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Price sensitivity of demand: Elasticity

relative rate of change of demand $\approx -E(p)$ (relative rate of change of price)
 percent change of demand $\approx -E(p)$ (percent change of price)

$$x=f(p) = 40,000-1000p$$

Found $E(p) = p/(40-p)$

$$E(8) = 8/(40-8)=8/32=1/4 = .25 < 1.$$

At $p=\$8$: a price increase of 10% will create a demand decrease of $.25(10\%)=2.5\%$

$E(30)$

$E(20)$

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Price sensitivity of demand: Elasticity

E(p)	Demand	Interpretation
$0 < E(p) < 1$	Inelastic	
$E(p) > 1$	Elastic	
$E(p) = 1$	Unit	

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Revenue and elasticity of demand

If we have a linear function relating price to demand,

$$x = f(p)$$

can we express revenue as a function of price???

$$R(p) = (\text{quantity})(\text{price})$$

Find

$$R'(p) =$$

$$\begin{aligned} R \text{ increasing} &\Leftrightarrow R'(p) > 0 \Rightarrow E(p) < 1 \Leftrightarrow \text{Demand is inelastic} \\ R \text{ decreasing} &\Leftrightarrow \end{aligned}$$

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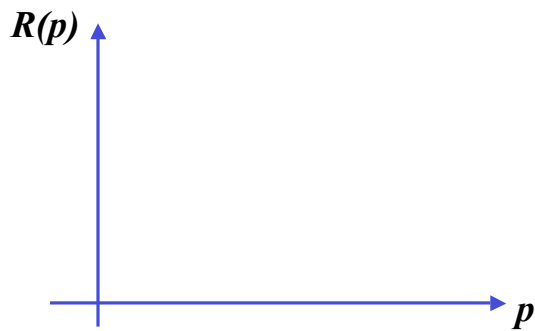
Revenue and elasticity of demand

$$R \text{ increasing} \Leftrightarrow R'(p) > 0 \Rightarrow E(p) < 1 \Leftrightarrow \text{Demand is inelastic}$$

So

% Increase price \Rightarrow smaller % decrease in demand \Rightarrow increased revenue.

$$R \text{ decreasing} \Leftrightarrow R'(p) < 0 \Rightarrow E(p) > 1 \Leftrightarrow \text{Demand is elastic}$$



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Application

A sunglass manufacturer currently sells one type of sunglasses for \$10 per pair. The price demand function is: $x=f(p)=7000-500p$.

If the current price is increased, will revenue increase or decrease?

$E(p) =$