WARM UP EXERCSE

A cable company has found that the total number N (in thousands) of subscribers t months after the installation of the system is given by

$$N(t) = 200t / (t + 5)$$

Find N (15) and N ‘ (15). Write an interpretation of these results.

§11.4 Chain Rule: Power Form

The student will learn about:

- **the easy version of the chain rule,**
- **combining different rules of derivation**
- **application**

Some examples

$$\frac{d}{dx}\left(5x^3\right)^2 = \frac{d}{dx}\left(125x^6\right) =$$

$$\frac{d}{dx}\left(5x^3\right)^2 = \frac{d}{dx}\left(\left(5x^3\right)\left(5x^3\right)\right) =$$

$$\frac{d}{dx}\left(5x^3\right)^3 = \frac{d}{dx}\left(\left(5x^3\right)^2\left(5x^3\right)\right) =$$

Some examples

$$\frac{d}{dx}\left(u(x)\right)^2 = \frac{d}{dx}\Big(\left(u(x)\right)\left(u(x)\right)\Big) =$$

$$\frac{d}{dx}\left(u(x)\right)^3 = \frac{d}{dx}\Big(\left(u(x)\right)^2\left(u(x)\right)\Big) =$$

Chain Rule: Power Rule.

Theorem 1. (General Power Rule or easy Chain Rule) If u (x) is a differential function, n is any real number, and

$$f(x) = [u(x)]^n$$

then $$f'(x) = n[u(x)]^{n-1} u'(x)$$

$$= n\,u^{n-1}u'$$

or $$\frac{d}{dx}u^n = n\,u^{n-1}\frac{du}{dx}$$

Find the derivative of $y = (x^3 + 2)^5$.

Example 2

Find the derivative of $y = \sqrt{x^3 + 3}$

Example 3: Combining Rules of Differentiation

Find $f'(x)$ if $f(x) = \dfrac{x^4}{(3x-8)^2}$.

Example 4

Let $f(x) = x^2 (1 - x)^4$; *at* $x = 2$.

Find $f'(x)$ and find the equation of the line tangent to the graph of f at the indicated value of x.

Application

The number x of stereo speakers people are willing to buy per week at a price of $p is given by

$$x = 1{,}000 - 60\sqrt{p + 25} \qquad \text{for } 20 \le p \le 100$$

1. Find dx/dp.

2. Find the demand and the instantaneous rate of change of demand with respect to price when the price is $75.

Application continued

The number x of stereo speakers people are willing to buy per week at a price of $p is given by

$$x = 1{,}000 - 60\sqrt{p + 25} \qquad \text{for } 20 \le p \le 100$$

3. Give a verbal interpretation of these results.

With $f(75) = 400$ and $f'(75) = -3$ that means

Summary.

If

$$y = f(x) = [u(x)]^n$$

then

$$\frac{d}{dx} u^n = n u^{n-1} \frac{du}{dx}$$